Mem. S.A.It. Vol. 82, 867

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The long-term azimuthal structure of the Galactic Cosmic Ray proton distribution due to anisotropic diffusion

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Abstract. In the description of cosmic ray transport based on a diffusion-convection equation, the spatial diffusion of energetic particles, in general, has to be treated by employing a tensorial quantity, i.e. using different diffusion strengths along and perpendicular to the magnetic field, respectively. This leads to results for the distribution function of these particles which are different from those obtained with just a scalar diffusion coefficient, i.e. isotropic diffusion. Since the sources of cosmic rays are supposedly mainly supernova remnants distributed along the spiral arms of the galaxy, our solar system experiences different levels of cosmic ray intensities along its way around the galactic center. The actual azimuthal structure of the distribution then depends critically on the diffusion model and the employed magnetic field model. The resulting variations are, apart from their general astrophysical scope, also of interest in the context of (very) long-term climatology.

Key words. Cosmic Rays – ISM: magnetic fields – Diffusion – Astroparticle physics

1. Introduction

During its orbit around the galactic center, the solar system experiences varying conditions of its surrounding interstellar medium (ISM). On the timescale of hundreds of millions of years, the sun and the earth leave and enter the galactic spiral arms and may thus be subject to variations in the cosmic ray (CR) flux with about the same frequency. This is mainly due to the most probable sources of CRs, i.e. supernova remnants, being much more common inside the arms, than in the interarm regions. A possible connection between long-term climate variations and the associated spiral arm passages has been proposed by Shaviv (2002). To evaluate this connection and improve our understanding of CR fluxes during the history of our solar system, accurate models for the CR transport and its resulting azimuthal variation in the galaxy are needed. The description of CR transport is usually done by invoking a diffusion-convection equation for the particle distribution function $f(\boldsymbol{r}, p, t)$ of the following form:

$$\frac{\partial f}{\partial t} = \nabla \cdot (\hat{\kappa} \nabla f) + p a_\pi \frac{\partial f}{\partial p} + 3 a_\pi f + \frac{q}{p^2} \qquad (1)$$

Send offprint requests to: F. Effenberger see e.g. the review by Strong et al. (2007). Here $\boldsymbol{r}$ and p describe the location in space and (isotropic) momentum, respectivly. a_π denotes the energy loss processes due to pion production, which in this case is assumed to be linear in momentum for protons in the energy range between 1 – 100 GeV (Mannheim & Schlickeiser, 1994). A possible contribution from a galactic wind has been neglected so far in our model, but could be included in further studies without major complications. An estimate from simple 1-D galactic wind models (e.g. Zirakashvili et al., 1996) shows that the influence of adiabatic losses from such a wind is at most of the order of the pion losses. The notation $\hat{\kappa}$ indicates that the diffusion part of the equation, in general, has to be described by a tensor, i.e. a 3x3 matrix for the spatial diffusion. The need to distinguish between a diffusion parallel and a different diffusion perpendicular to the magnetic field is a long standing paradigm in the description of heliospheric CR transport (see e.g. Fichtner, 2005). There are, however, strong indications that anisotropic diffusion is also of significance to galactic propagation, both from numerical experiments (concerining the galactic dynamo, see e.g. Hanasz et al., 2009) and from basic theory (Shalchi et al., 2010). Both studies claim that there is at least an anisotropy of a factor of ten, i.e the perpendicular diffusion coefficient $\kappa_\perp$ is equal to 0.1 times the parallel diffusion coefficient $\kappa_\parallel$. In the following, the specific assumptions of our model regarding the sources of CRs and the diffusion tensor are briefly outlined. A short description of the numerical solution method to the transport equation with stochastical differential equations is given. Finally, the results are presented and some conclusions are drawn.

2. The model and method of solution

2.1. Source distribution

We assume a source distribution which follows the galactic spiral arm model established by Vallée (2002), which consists of four logarithmic and symmetric arms (see Fig. 1 for an illustration). Around these, we take a gaussian shape in galactic cylindrical coordinates $[r, \phi, z]$ (analogous to the approach in Shaviv, 2003) to yield an analytic expression for the source term q, by summing up over all four arms ($n \in \{1,2,3,4\}$):

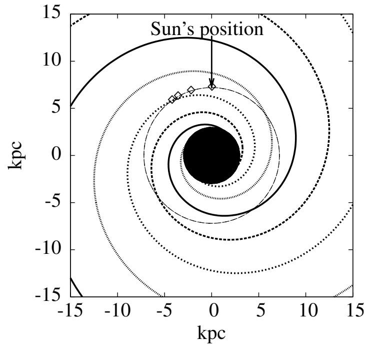

Fig. 1. Orientation of the spiral arms in the galaxy, constituting the CR sources. The dashed line shows the sun's orbit across the spiral arms and the diamonds indicate the positions, at which the CR spectra in Fig. 2 are calculated.

$$q_n = p^{-s} \exp\left(-\frac{(r-r_n)^2}{2\sigma_r^2} - \frac{z^2}{2\sigma_z^2}\right) \qquad (2)$$

with $r_n = r_0 \exp(k(\phi + \phi_n))$. ϕ_n introduces the symmetric rotation of each arm by 90°, i.e. $\phi_n = (n-1)\pi/2$. $k = \cos\psi$ with $\psi = 12°$ is the constant pitch-angle cosine of the spiral arms. We take $\sigma_r = \sigma_z = 0.2$ kpc to have a reasonable interarm separation, while $r_0 = 2.3$ kpc according to Vallée's model. The spectral index s of the sources' power law injection in momentum is set to $s = 2.1$ and the galaxy has a radius of 15 kpc and a height of 4 kpc, all similar to Büsching & Potgieter (2008).

2.2. Diffusion tensor and magnetic field

The diffusion tensor is constructed from the assumption that it is diagonal in a frame of reference which is aligned to the magnetic field. This "local" tensor

$$\hat{\kappa}_L = \begin{pmatrix} \kappa_{\perp 1} & 0 & 0 \\ 0 & \kappa_{\perp 2} & 0 \\ 0 & 0 & \kappa_\parallel \end{pmatrix} \qquad (3)$$

is then transformed to the global frame, in which the magnetic field is expressed, by a ma-

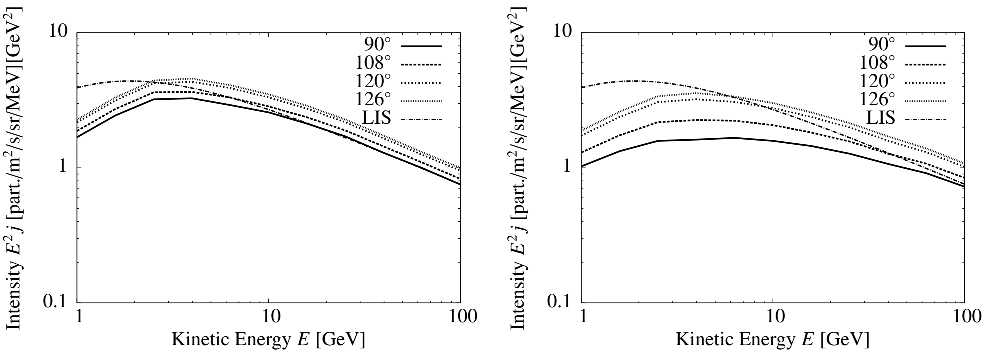

Fig. 2. Proton spectra for different positions along the sun's orbit (see Fig. 1 for a sketch of the respective locations). The left panel shows the results for isotropic diffusion, i.e. $\kappa_\perp = \kappa_\parallel$ while the right panel shows results from a similar calculation but with anisotropic diffusion, i.e. $\kappa_\perp = 0.1\kappa_\parallel$. The results for anisotropic diffusion were renormalized with a relative factor of 0.154 to compensate for the higher total flux due to the stronger confinement of CRs in the disk. The dash-dotted line indicates the Local Interstellar Spectrum (LIS) from Reinecke et al. (1993) for comparison.

trix A with the usual matrix transformation rule $\hat{\kappa} = A\hat{\kappa}_L A^T$. The columns of the transformation matrix A consist of the three unit vectors of the local frame, expressed in the global frame. For the simple case of a magnetic spiral field, which is aligned to the spiral arm structure and which has no z-component, the unit-vectors are $\mathbf{e}_1 = \mathbf{e}_z \times \mathbf{t}, \quad \mathbf{e}_2 = \mathbf{e}_z, \quad \mathbf{e}_3 = \mathbf{t}$. Here, $\mathbf{t} = \frac{\mathbf{B}}{|\mathbf{B}|}$ is the tangential unit vector of the magnetic field, for which we make the ansatz:

$$\mathbf{B} = (\sin\psi\,\mathbf{e}_r + \cos\psi\,\mathbf{e}_\phi)\frac{1}{r}\exp\left(-\frac{z^2}{2\sigma_z^2}\right) \quad (4)$$

The diffusion coefficient $\kappa_\parallel$ is taken to have the same broken power law dependence as the scalar diffusion coefficient in Büsching & Potgieter (2008), namely:

$$\kappa_\parallel = \kappa_0 \left(\frac{p}{p_0}\right)^\alpha \quad (5)$$

with $\alpha = 0.6$ for $p > p_0$, $\alpha = -0.48$ for $p \leq p_0$, $\kappa_0 = 0.027\,\mathrm{kpc}^2/\mathrm{Myr}$ and $p_0 = 4\,\mathrm{GeV/c}$. This general form of the diffusion coefficient has recently been motivated from basic theory, see Shalchi & Büsching (2010). For the perpendicular diffusion coefficients, we take $\kappa_{\perp 1} = \kappa_{\perp 2} = 0.1\kappa_\parallel$, which is a reasonable upper limit value in the energy range of 1 – 100 GeV (see Fig. 7 in Shalchi et al., 2010).

2.3. The Method of Stochastical Differential Equations

The transport equation (1) is solved by applying a pseudo-particle tracing numerical method to an equivalent system of stochastic differential equations (SDEs) of the general form (Gardiner, 1994):

$$dx_i = A_i(x_i)ds + \sum_j B_{ij}(x_i)dW_i \quad (6)$$

We employ the time-backward Markov stochastic process, meaning that we trace back the pseudo-particles (representing phase space volumes) from a given phase-space point of interest, until they hit the integration boundary. For a general overview on the application of this method to CR propagation see Zhang (1999) and Farahat et al. (2008). The details of the actual numerical code applied in our case are discussed to some extent in Strauss et al. (2011) although the development of the code is still ongoing, especially in the application to galactic problems. Generally speaking, this method enables us to calculate every phase space point we are interested in, just by tracing back enough pseudo-particles, without any major restrictions due to numerical stability or memory issues, arising e.g. from high dimensionality.

3. Results

We calculated proton spectra and orbital flux variations for two parameter sets, differing only in the diffusion model, to evaluate the influence of anisotropic diffusion. The first one takes just the parallel diffusion coefficient described in section 2.2 for a scalar diffusion model. This is equivalent to setting $\kappa_{\perp 1} = \kappa_{\perp 2} = \kappa_{\|}$. The second case takes the above described anisotropic diffusion into account. The left panel of Fig. 2 shows the resulting proton spectra at different positions of the sun for isotropic diffusion. On can clearly see that they differ independently from energy and only by a small amount. For the anisotropic case in the right panel, however, these differences are much more pronounced and also energy dependent. They are largest for the middle to low energy range around a few GeV. This effect can be seen even more clearly in Fig. 3, where the orbital variation is plotted for 1 and 100 GeV. For the lower energy, the variation in the anisotropic case is more than 30% larger than for isotropic diffusion, while for the higher energies, this effect is much smaller, although still present. Note as well that the average total flux is much higher in the anisotropic case (which can be seen in the different normalization of both spectra), due to the stronger confinement in the disk, resulting from the lower perpendicular diffusion of $\kappa_{\perp 2}$ in the z-direction.

4. Conclusions

We calculated proton spectra for the anisotropic spatial diffusion of CRs in the galaxy. The spectral shape is in general agreement with the expected values from observation. The results show a significant enhancement of orbital flux variation for the solar orbit in the energy range of 1-10 GeV in comparison to a model of scalar diffusion. Thus, our results indicate that anisotropic diffusion is of importance for galactic CR propagation and might amplify climatic effects in the context of the proposed CR-climate connection.

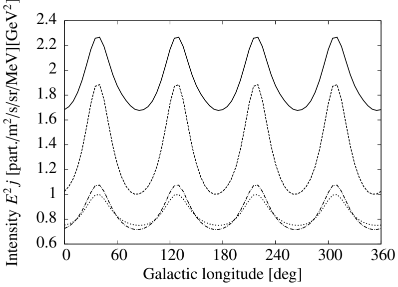

Fig. 3. Variation of the proton intensity along the sun's orbit at $r = 7.2$ kpc plotted against galactic longitude (see Fig. 1 for the orientation). The solid line shows 1 GeV protons for the case of isotropic diffusion while the dashed line is for anisotropic diffusion (same calculations as in Fig. 2). The dotted (isotropic) and dotted-dashed (anisotropic) lines show the same difference but for 100 GeV protons.

Acknowledgements. We thank A. Marcowith for giving us the possibility to present these results in the course of his workhop. The work was carried out within the 'Galactocauses' project (FI 706/9-1) funded by the Deutsche Forschungsgemeinschaft (DFG) and benefitted from the DFG-Forschergruppe 1048 (project FI 706/8-1) as well as from the 'Heliocauses' DFG-project (FI 706/6-3).

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